Nonlinear estimation and control for small-scale wind turbines

Pradeep Bhatta and Michael A. Paluszek
Princeton Satellite Systems, Inc.
33 Witherspoon St, Princeton, NJ 08542, USA
{pradeep,map}@psatellite.com

Abstract:
This paper presents application of nonlinear control and estimation methods for a variable speed wind energy conversion system (WECS). A low-order, nonlinear model that captures the dominant dynamics of the wind turbine drive-train is considered. An unscented Kalman filter (UKF) is used for online estimation of parameters. The control law, derived from an energy-like Lyapunov function, guarantees system stability for all operating points of the WECS. The structure of the control law provides physically intuitive guidelines for gain selection. Furthermore, the robustness properties inherent in all energy-based control approaches is preserved. Simulations of speed regulation and stiffness estimation are presented.

Keywords:
Small-Scale Wind Turbines, Energy-based control, Lyapunov function, Unscented Kalman Filter, Stability.

1 Introduction

While debates over the causes and consequences of global warming and climate change continue, there is growing consensus on the benefits of technologies that harvest renewable sources of energy. Advances in the areas of computing, communications, and manufacturing propel development of novel methods for achieving high extraction efficiencies that render renewable energy profitable in today's market. The key to this development is identification of advanced technologies across disciplines that can be beneficially transferred or adapted to renewable energy harvesting. In this paper we present an application of nonlinear control and estimation methods to a model of a small-scale wind energy conversion system. The motivation for this approach is the need for a versatile control strategy that can be implemented on relatively inexpensive systems.

There are typically three closely-coupled control objectives for a wind energy conversion system (WECS) [1]: (i) maximum power capture, (ii) mechanical load mitigation, and (iii) power quality protection. Capturing maximum power is a very important consideration for small-scale WECS operating in an environment with low winds. Transient and high-frequency must be alleviated to increase the life-span of the system, thereby reducing the total cost of generating wind power. Finally, WECS linked to the grid must adhere to certain interconnection standards.

To realize the control objectives, variable speed, variable pitch and active yaw control mechanisms can be employed. Variable speed wind turbines interface electronic converters between the generator and the AC grid, thereby decoupling the rotational speed from the grid frequency. The speed of the turbine is controlled by adjusting generator torque. Variable speed operation increases the energy capture at low wind speeds. On the other hand, variable pitch operation enables efficient power regulation at higher than rated winds. Having speed and pitch regulation allows realization of the ideal power curve [1]. Variable pitch operation also alleviates transient loads. Simultaneous control of pitch and speed above rated wind speed also provides important benefits to the dynamic performance of the WECS under high wind conditions. Yaw control is used to maximize wind energy capture by keeping the rotor of the turbine facing towards the wind.

For the purpose of control design, low-order mathematical models that capture the dominant system characteristics of the wind turbine will be used. Such models are analytically tractable, and amenable to application of tools from dynamical systems and control theory. Robust controllers account for modeling uncertainties.

Variable speed and variable pitch control methods have been very popular [2]-[5]. Control systems for many large-scale WECS are based on gain-scheduling techniques in which the nonlinear model is linearized about a selected set of operating points, and a linear controller is designed for each of these linear plants. Further a switching algorithm is determined. Stability, robustness and performance properties of the system cannot be assessed based on the feedback properties of the family of linear plants. Some of these issues have been resolved by advanced linear control methods such as [6].

We consider a Lyapunov-based control law for the nonlinear system model. The nonlinear controller can be tuned based on system performance priorities, with guaranteed stability. The structure of the control law provides physically intuitive guidelines for gain selection. Furthermore, the robustness properties inherent in all energy-based control approaches is preserved. Below we present a Lyapunov-based control law for regulating the tip-speed ratio to enable maximum power tracking for a variable-speed fixed-pitch WECS.

2 Lyapunov-based drive train control for a variable-speed fixed-pitch WECS

The aerodynamic power P_a captured by the rotor of the wind turbine is usually given by the expression

$$P_a = \frac{1}{2}\rho\pi R^2 C_P(\lambda) V^3, \tag{1}$$

where ρ is the air density, R is the rotor radius, C_P is the power coefficient, v is the effective wind speed, and $\lambda = \frac{\Omega_r R}{V}$ is the tip-speed ratio, where Ω_r is the rotor angular speed. We note that the power coefficient is a function only of λ, since the pitch of the turbine is held constant in our example. Correspondingly the torque in the rotor T_r is expressed in terms of the torque coefficient $C_Q = C_P/\lambda$:

$$T_r = \frac{1}{2}\rho\pi R^2 C_Q(\lambda)V^2. \tag{2}$$

The torque coefficient takes a maximum value $C_{Q_{max}}$ at a certain optimum tip-speed ration λ_{max}. Typically just a discrete set of values for the torque coefficient is available. A good approximation of the torque coefficient that is commonly employed [1] is a second-order polynomial of the form:

$$C_Q(\lambda) = C_{Q_{max}} - K_Q(\lambda - \lambda_{Q_{max}})^2, \tag{3}$$

where $K_Q > 0$ is a constant.

The dynamic model of the wind turbine drive-train can be described by the following equations [1]:

$$\begin{bmatrix} \dot{\theta}_s \\ \dot{\Omega}_r \\ \dot{\Omega}_g \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ -\frac{K_s}{J_r} & -\frac{B_s}{J_r} & \frac{B_s}{J_r} \\ \frac{K_s}{J_g} & \frac{B_s}{J_g} & -\frac{B_s}{J_g} \end{bmatrix} \cdot \begin{bmatrix} \theta_s \\ \Omega_r \\ \Omega_g \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{T_r}{J_r} \\ -\frac{T_g}{J_g} \end{bmatrix}, \tag{4}$$

where θ_s is the torsion angle depicting the difference in angular positions of the rotor and generator, Ω_g is the angular speed of the generator, T_g is the generator torque, K_s and B_s are the stiffness and damping of the transmission respectively, J_r and J_g are the moments of inertia of the rotor and generator respectively. For a variable-speed fixed-pitch turbine, T_g regulated by the power generation unit can be considered to be a control input to the above drive-train dynamics.

The goal of the drive-train control law is to follow a control strategy, specified as a locus of operating points. The operating points are equilibria of the system modeled by equation (4). The control strategy, usually chosen based on trade-offs between energy extraction and load alleviation, picks a desired steady rotor speed $\Omega_{r,e}$ for each steady wind speed V_e, thereby effectively determining the steady operating torque $T_{r,e}$. From the first component of equation (4), the steady generator speed $\Omega_{g,e} = \Omega_{r,e}$. The corresponding torsion angle $\theta_{s,e}$ and generator torque $T_{g,e}$ can be calculated by solving the equilibrium relations corresponding to the last two components of equation (4).

For the purpose of deriving the control law let us rewrite equation (4) in terms of deviations from the operating point:

$$\begin{bmatrix} \dot{\bar{\theta}}_s \\ \dot{\bar{\Omega}}_r \\ \dot{\bar{\Omega}}_g \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ -\frac{K_s}{J_r} & -\frac{B_s}{J_r} & \frac{B_s}{J_r} \\ \frac{K_s}{J_g} & \frac{B_s}{J_g} & -\frac{B_s}{J_g} \end{bmatrix} \cdot \begin{bmatrix} \bar{\theta}_s \\ \bar{\Omega}_r \\ \bar{\Omega}_g \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\bar{T}_r}{J_r} \\ -\frac{\bar{T}_g}{J_g} \end{bmatrix}, \tag{5}$$

where $\bar{(.)}$ denotes the variable minus its value at the operating point. The equilibrium (operating point) in terms of the new variables is the origin $(0, 0, 0)$.

Using equations (2)-(3) and the definition of λ we can write

$$\bar{T}_r = -K_1\bar{\Omega}_r^2 + K_2\bar{\Omega}_r, \tag{6}$$

where K_1, K_2 are constants.

In order to compute the control law ($\bar{T}_g$) and prove the stability of the closed-loop system, consider the Lyapunov function candidate,

$$\Phi = \frac{1}{2}\left[\left(\frac{2J_r + J_g}{J_r}\right)K_s\bar{\theta}_s^2 + J_g\left(\bar{\Omega}_r - \bar{\Omega}_g\right)^2 + J_r\bar{\Omega}_r^2 + J_g\bar{\Omega}_g^2\right]. \tag{7}$$

It is straightforward to verify that Φ is a valid Lyapunov function candidate [7]. Now, we compute

$$\dot{\Phi} = -B_s\left[\left(\frac{J_r + J_g}{J_r}\right)\left(\bar{\Omega}_r - \bar{\Omega}_g\right)^2 + \bar{\Omega}_r^2 + \bar{\Omega}_g^2\right] + \frac{J_g + J_r}{J_r}\bar{\Omega}_r^2\left(K_2 - K_1\bar{\Omega}_r\right)$$
$$-\frac{J_g}{J_r}\bar{\Omega}_r\bar{\Omega}_g\left(K_2 - K_1\bar{\Omega}_r\right) - \bar{T}_g\left(\bar{\Omega}_r - 2\bar{\Omega}_g\right). \tag{8}$$

We chose a control law of the form

$$\bar{T}_g = m\bar{\Omega}_r + n\bar{\Omega}_g, \tag{9}$$

where m and n are constants. Further, we set

$$n - 2m = \left(1 + \frac{J_g}{J_r}\right)K_2. \tag{10}$$

Substituting the inequalities $-\bar{\Omega}_r^3 \leq \frac{1}{2}\left(\bar{\Omega}_r^4 + \bar{\Omega}_r^2\right)$, and $\bar{\Omega}_g\bar{\Omega}_r^2 \leq \frac{1}{2}\left(\bar{\Omega}_r^4 + \bar{\Omega}_g^2\right)$ in equation (8) we get the following relation,

$$\dot{\Phi} \leq -a\bar{\Omega}_r^2 - b\bar{\Omega}_g^2 - c\left(\bar{\Omega}_r - \bar{\Omega}_g\right)^2, \tag{11}$$

where,

$$a = m + B_s - \frac{J_g + J_r}{J_r}\left(\frac{K_1}{2} + K_2\right) - \frac{2J_g + J_r}{2J_r}K_1\bar{\Omega}_r^2 \tag{12}$$

$$b = 2m + B_s - \frac{J_g}{2J_r}\left(2K_2 + K_1\right) \tag{13}$$

$$c = B_s\left(\frac{J_r + J_g}{J_r}\right). \tag{14}$$

From the above equations we see $c > 0$, and if we pick m large enough we also have $a, b > 0$ for some $\left|\bar{\Omega}_r\right| \leq A$, where A is some constant. Thus,

$$\dot{\Phi} \leq 0 \tag{15}$$

for all $x := \left(\bar{\theta}_s, \bar{\Omega}_r, \bar{\Omega}_g\right) \in B_A$, where B_A is a ball of radius A in the space $S^1 \times \mathbb{R}^2$. This

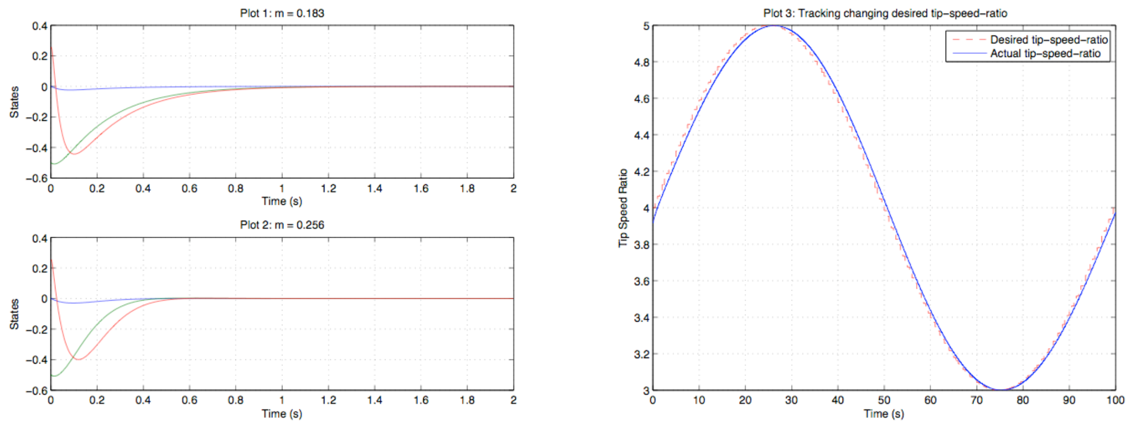

Figure 1: Closed-loop simulations for a variable-speed, fixed-pitch WECS

proves local asymptotic stability of the closed-loop system with the region of attraction given by B_A. In fact, the control law also guarantees local exponential stability. Figure 2 demonstrates the application of the control law in a drive-train dynamics simulation. Plots 1 and 2 of the figure show the convergence of the states of the system to the desired operating point (the origin in both cases) for two different values of the control gain. The larger m provides faster convergence. Plot 3 shows the result of a simulation where the system is made to track a desired tip-speed ratio profile. Good performance is observed in all simulations.

We note that the size of the region of attraction (determined by A) is a function of the adjustable control gain m. Choosing a larger value of m will lead to a faster transient and larger region of attraction. However, a large m may not be desirable if the wind has dominant high frequency components, since this may lead to larger mechanical loads.

The control law given by equation (9) holds for all operating points of the wind turbine. Furthermore, the Lyapunov-based control law is robust with respect to model uncertainties. We also have a systematic procedure for determining the speed of convergence and guaranteed region of attraction. The above theoretical results can be extended to varying wind-speed situations. In the case of a variable-speed, variable-pitch WECS, similar control laws can be derived after incorporating additional dynamics due to pitch regulation.

3 Nonlinear Estimation

The unscented Kalman filter (UKF) can be used to calculate accurate state and parameter estimates of the wind energy conversion system (WECS). The UKF removes some of the shortcomings of the extended Kalman filter (EKF) (the most commonly used estimation method for nonlinear systems till recently) by using the unscented transformation (UT). Unlike the EKF, the UKF does not require any derivatives or Jacobians of either the state equations or measurement equations. Instead of just propagating the state, the filter propagates a set of sample, or sigma, points which are determined from the a priori mean and covariance of the state. The sigma points undergo the unscented transformation. Then the posterior mean and covariance of the state are determined from the transformed sigma points.

Since UKF can accurately estimate the nonlinearities of a system, it is a very attractive tool for WECS control implementation and parameter estimation. The aerodynamical forcing driving the wind turbine has a highly nonlinear dependence on the blade pitch, rotor speed and wind speed. Very often the wind speed measurement is not accurate or absent. Moreover, while the wind speed distribution across the rotor blade determines the actual aerodynamic forcing, wind measurements are made from only one or few positions on the wind turbine system, and the drive-train dynamical model only considers an effective wind speed. Using a UKF estimation scheme will provide accurate estimates of the effective wind speed. The UKF can also be used to estimate other hard to measure signals such as rotor torque, as well as to estimate the parameters of the system. Having an on-board parameter estimation capability will be particularly useful in mass-produced wind turbine systems.

A summary of UKF equations can be found in [8]. The implementation of UKF follows a systematic procedure as described in [8,9]. This procedure for parameter estimation is summarized below:

Let w represent the vector of unknown parameters of the model of the wind energy conversion system. The UKF is initialized with the following parameter estimate and covariance:

$$\hat{w}_0 = E[w], \quad P_{w_0} = E[(w - \hat{w}_0)(w - \hat{w}_0)^T]. \tag{16}$$

Let L be the total number of states in the system. At any time-step k, $2L + 1$ a priori sigma points are determined:

$$\Xi_{k-1} = \left[\; \hat{x}_{k-1} \;\; \hat{x}_{k-1} + \gamma\sqrt{P_{k-1}} \;\; \hat{x}_{k-1} - \gamma\sqrt{P_{k-1}} \; \right], \tag{17}$$

where $\gamma = \alpha\sqrt{L+\kappa}$, α and κ are adjustable parameters of the filter, discussed further after equation (25), and

$$\hat{w}_k^- = \hat{w}_{k-1} \tag{18}$$

$$P_{w_k}^- = P_{w_{k-1}} + R_{k-1}^r. \tag{19}$$

The next step in implementing UKF involves obtaining an update of the estimated measurement vector $\hat{d}_k$. This can be done as follows:

$$\hat{d}_k = \sum_{i=0}^{2L} W_i^{(m)} \mathcal{D}_{i,k|k-1}, \tag{20}$$

where $\mathcal{D}_{i,k|k-1}$ represents the updated measurement estimate vector corresponding to the i^{th} sigma point:

$$\mathcal{D}_{k|k-1} = G[x_k, \Xi_{k|k-1}] \tag{21}$$

In equation (21), G represents the nonlinear mapping between the measured outputs and the system parameters. $W_i^{(m)}$ and $W_i^{(c)}$ are weights given by

$$W_0^{(m)} = \frac{\lambda}{L+\lambda} \tag{22}$$

$$W_0^{(c)} = \frac{\lambda}{L+\lambda} + 1 - \alpha^2 + \beta \tag{23}$$

$$W_i^{(m)} = W_i^{(c)} = \frac{1}{2(L+\lambda)} \qquad i = 1, \ldots, 2L \tag{24}$$

and,

$$\lambda = \alpha^2(L+\kappa) - L. \tag{25}$$

In the above equations α, β and κ are adjustable parameters of the filter. The parameter α determines the spread of sigma points around the parameter estimate, and is usually set to $1e-3$. The parameter κ also influences scaling, and the parameter β incorporates prior knowledge of the distribution of x. For gaussian distributions, β is set to 2.

Ξ_{k-1} and $\mathcal{D}_{i,k|k-1}$ are used to compute the measurement covariance, $P_{d_k d_k}$, and the cross-correlation covariance, $P_{w_k d_k}$:

$$P_{d_k d_k} = \sum_{i=0}^{2L} W_i^{(c)} [\mathcal{D}_{i,k|k-1} - \hat{d}_k][\mathcal{D}_{i,k|k-1} - \hat{d}_k]^T + R_k^e \tag{26}$$

$$P_{w_k d_k} = \sum_{i=0}^{2L} W_i^{(c)} [\Xi_{i,k|k-1} - \hat{w}_k][\mathcal{D}_{i,k|k-1} - \hat{d}_k]^T, \tag{27}$$

where R_k^e is the measurement covariance matrix. The Kalman gain matrix is

$$K_{x_k} = P_{w_k d_k} P_{d_k d_k}^{-1}. \tag{28}$$

Finally, the measurement update equations are used to determine the mean parameter estimate, $\hat{w}_k$, and the covariance, P_{w_k}:

$$\hat{w}_k = \hat{w}_k^- + K_k(d_k - \hat{d}_k^-) \tag{29}$$

$$P_{w_k} = P_{w_k}^- - K_k P_{d_k d_k} K_k^T. \tag{30}$$

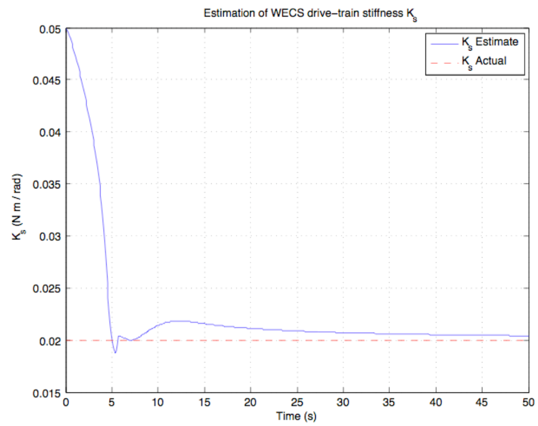

Figure 2: Parameter estimation using UKF

Figure 2 presents a numerical example of estimating the stiffness K_s of the wind energy conversion system model using the UKF. In this example the UKF is initialized with a K_s estimate of 5 Nm/rad, while the actual $K_s = 0.2$ Nm/rad. Generator speed Ω_g measurements are used in the UKF to derive estimates of K_s. All other parameters are considered to be known accurately. The parameter estimation is very robust with respect to the initial estimate of K_s. In the simulation presented the initial estimation error was 150 %.

4 Conclusions

Nonlinear control and estimation methods have been applied to a wind energy conversion system model. These methods can be implemented systematically and have proven convergence properties. Their inherent robustness and convergence properties renders development and integration of advanced control and estimation algorithms into small-scale systems affordable.

References

[1] Bianchi, F. D., De Battista, H. and Mantz, R. J., *Wind Turbine Control Systems: Principles, Modelling and Gain Scheduling Design.* Springer (2006).

[2] Datta, R. and Ranganathan, V. T., A method of tracking peak power points for a variable speed wind energy conversion system. *IEEE Transactions on Energy Conversion* **18** (1), 163–168 (2003).

[3] Tan, K. and Islam, S., Optimal control strategies in energy conversion of PMSG wind turbine systems without mechanical sensors. *IEEE Transactions on Energy Conversion* **19** (2), 392–399 (2004).

[4] Boukhezzar, B. and Siguerdidjane, H., Nonlinear control of variable speed wind turbines for power regulation. *Proceedings IEEE Conference on Control Applications* (2005).

[5] Akhmatov, V., Modeling and ride-through capability of variable speed wind turbines wit permanent magnet generators. *Wind Energy* **9** (4) 313–326 (2006).

[6] Bianchi, F.D., Mantz, R. J. and Christiansen, C. F., Control of variable-speed wind turbines by LPV gain scheduling. *Wind Energy* **7** (1), 1–8 (2004).

[7] Khalil, H. K., *Nonlinear Systems*, Prentice Hall, Upper Saddle River, NJ, (2003).

[8] Wan, E. A. and van der Merwe, R., The unscented Kalman filter for nonlinear estimation. *Proc. IEEE Adaptive Systems for Signal Processing, Communications, and Control Symposium* pp 153–158 (2000).

[9] VanDyke, M. C., Schwartz, J. L. and Hall, C. D., Unscented Kalman FIltering for Spacecraft Attitude State and Parameter Estimation. *Advances in Astronautical Sciences* (2005).