1 Introduction

Counting probability problems mean that you can count all outcomes. For example, if you flip a coin it can be either heads or tails. That means flipping a coin has 2 outcomes. You are equally likely to flip a heads or a tales so we say they have the same probability. Since the only possible outcomes are a heads or a tails we say that probability of a heads or a tails is 1. Since the probabilities are equal the probability of a heads is 1/2 and the probability of a tales is 1/2.

Suppose we roll a dice. A standard dice has six sides with numbers 1 through 6. The probability of getting any number is 1/6. The probability of getting some number (1 through 6) is 1.

If we roll the dice once or flip the coin once we have one event. Suppose with have multiple events? For example, suppose with flip the coin twice? Here are all possible outcomes.

HH

HT

TH

TT

where the first letter is event 1 and the second is event 2. Since these are the only possible outcomes of two events the sum of the probabilities of all of the outcomes must be 1. What is the probability of HH? To find it you multiple the probability of a heads time the probability of a heads which is

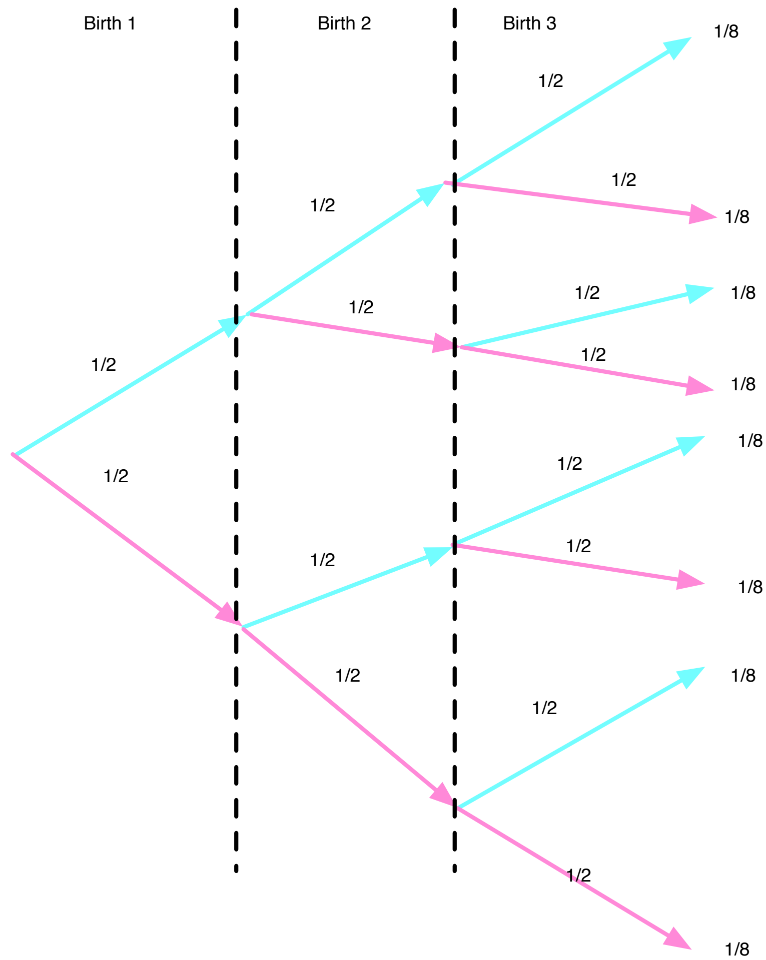
$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \tag{1-1}$$

2 Problems

2.1 Problem 1

> In a family three children are born. What is the probability that all are girls? What is the probability that one is a boy? What is the probability of 2 girls?

Each birth is an event and we can assume that boys and girl births are equally likely. An easy way to visualize this is with a tree diagram.



Each branch is a outcome of the event and the result is indicated by the color. The first birth is the first event and the result may be a boy or a girl with equal probability of 1/2. Remember, the sum of all of the probabilities of each event is 1. The second birth is event tow. We must add this branch to each of the event 1 branches. We repeat this with event 3. We label each branch with the probability which is always 1/2. On the right hand side with have all 8 possible outcomes. To find the probability of each outcome just multiply the probabilities on each branch. For example the probability of there being 3 girls is

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} \tag{2-2}$$

All outcomes have the same probability of 1/8 and the sum is 1. Only one branch is all pink meaning all girls so the probability of the family being all girls is 1/8. The probability of there being at least one boy is 1 minus the probability of there being 3 girls or 7/8. To find the probability of two girls add up the probabilities of all branches with only 2 pink lines. There are 3 paths with two pink lines so the probability is 3/8 since each path has the same probability of 1/8.

Compare this problem with the coin flip problem. That problem only had two events, the two coin flips. This problem had 3 events.

2.2 Problem 2

There a three boxes. Box A has 2 green and one blue ball. Box B has two red and one green ball. Box C has 1 green and 2 black. You first draw a ball from A, then B and then C. What is the probability of drawing 3 green balls?

The 3 events are

Draw from Box A

Draw from Box B

Draw from Box C

While all boxes have some green balls they each have different colors. If we think about drawing from box A we see that the probability of drawing a green ball is 2/3 and the probability of drawing a blue ball is 1/3. The sum of the probabilities is 1 as it should be. Each branch therefore has different probabilities. Even 2 is the draw from box A which may be either green or red balls. Even 3 is the draw from box C. To find the probability of each branch we multiply the numbers on each branch. for example the probability of the all green branch is 2/27. The sum of the probabilities of all branches is one. Of course 2/27 is the answer to the problem!

This problem, like the previous, had only two outcomes per event. In this case the probabilities of each outcome were different and the type of outcome (color of the ball) was different for each event. You can follow this same procedure with events with more than two outcomes but as you can imagine, the trees get really complicated!