



Learning Algebra, Trigonometry and Calculus Through Physics For Fourth through Eighth Graders

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CONTENTS

Contents	iii
1 Introduction	1
2 Newton's Law	3
3 The Bathroom Scale	7
4 Answers to Problems	11

CHAPTER 1

INTRODUCTION

Anyone can learn algebra, trigonometry and calculus. What are these subjects?

- Algebra is a branch of mathematics that uses symbols to represent numbers. In algebra you learn how to manipulate these symbols to solve problems.
- Trigonometry is a branch of mathematics that deals with the relationships between the sides and angles of triangles
- Calculus is a branch of mathematics that deals with the integration (adding) and differentiation (subtracting) of quantities

Why have these subjects scared students (and adults) for centuries? We contend it is because they are taught as “math” totally divorced from peoples’ experiences. When a student is given the rule

$$a(b + c) = ab + ac \tag{1-1}$$

it is entirely abstract and has no obvious meaning. A student views it as something to be memorized. Unfortunately, it is often quickly forgotten.

This book teaches algebra and calculus through teaching physics. Physics is the science of matter and energy and the interactions between the two. Math was developed largely to solve real-world problems. This book teaches math entirely through physical examples. For example, students are first exposed to algebraic manipulations through Newton’s first law

$$F = ma \tag{1-2}$$

This equation and its various forms are related to the students every day experiences. In subsequent chapters we expand into more complex examples. Each chapter is very short and covers one physical situation and a specific topic in algebra, trigonometry or calculus.

At the end of this course a student will be able to do solve any high school level algebra or trigonometry problem and will have the equivalent to a first course in mechanics. The student

will become comfortable with manipulating, reading and solving equations. The student will also understand basic calculus.

In this book we will provide “real-world examples” from:

The Consolidated Mechanisms Corporation with Chief Engineer Kate.

The World Wide Aerospace Company with Chief Scientist Pavan.

The Hoboken Ballet Theatre with Principal Dancers Pedro and Akina.

This book is a work in progress and is available for free. While we retain all copyrights you are free to download and distribute it as you like. Feedback is welcome, particularly feedback from students! Emails should be sent to algebra@psatellite.com.

Thanks in advance for reading our book. We hope you find it useful!

CHAPTER 2

NEWTON'S LAW

We start with equations. All equations have a left hand side, a right hand side and an equal sign.
Or

left-hand-side = right-hand-side

This means that everything on the left hand side must equal everything on the right hand side of the equation. For example

$$1 = 1 \quad (2-1)$$

is certainly true. This equation says that 1 (on the left hand side) equals 1 (on the right hand side). We also know that

$$837 = 837 \quad (2-2)$$

We could write any number of these equations. Algebra saves us time. We can just write

$$a = a \quad (2-3)$$

and say that a can represent ANY number. Our equation says that every number equals itself. Let's now move on to physics.

A very famous equation, Newton's law, is one of the most basic laws of physics. It is written as

$$F = ma \quad (2-4)$$

which is read as

force equals the mass of an object times the acceleration of an object

Sometimes famous equations are called laws. It is important to know the units of each symbol. In this case F is measured in Newtons, m in kilograms and a in meters per second squared. Don't

worry about what they mean, we'll explain that in future chapters. Its good to know how to write units. Meters per second is written as

$$m/s \tag{2-5}$$

Meters per second squared as

$$m/s^2 \tag{2-6}$$

² means squared which is simply

$$a^2 = a \times a \tag{2-7}$$

so it is a shorthand notation. We use the letters “kg” for kilogram and “s” for seconds. We write Newtons as “N”.

Each letter is a symbol and represents something physical. We use symbols so we don't have to write down the equation every time we change the numbers. So this equation represents

$$10 = 2 \times 5 \tag{2-8}$$

and

$$32 = 16 \times 2 \tag{2-9}$$

We never use more than one letter for a symbol. Also notice that we don't use a multiplication sign! We could have written

$$F = m \times a \tag{2-10}$$

So whenever we see ma we know it means $m \times a$. This just saves us some writing.

Acceleration means to change the speed. When your mom pushes on the accelerator pedal of your car the speed of the car changes as you can see by looking at the speedometer. This is because the car's engine creates a force that accelerates the car. When you start running you accelerate from zero speed to the speed at which you are running. In this case your muscles produce the force. When you drop an object and it falls to the ground gravity accelerates the object. In this case the gravity produces the needed force to accelerate the object towards the ground.

We could just as easily have written this equation as

$$F = am \tag{2-11}$$

which is read as

force equals the acceleration of an object times the mass of an object

We could also flip it around and say

$$ma = F \tag{2-12}$$

which is read as

the acceleration of an object times the mass of an object equals the force

This is our first algebraic fact! If two symbols are multiplied you can rearrange them so

$$am = ma \quad (2-13)$$

You know this from your multiplication tables. For example

$$5 \times 2 = 2 \times 5 \quad (2-14)$$

This equation is very useful as it stands. It allows us to calculate the force need to accelerate any object. Suppose, however, we have measured the force and the mass and want to find the acceleration? We can find the equation for the acceleration by rearranging the symbols.

We start with Newton's equation.

$$F = ma \quad (2-15)$$

Now, divide both sides by m . Our equation now looks like

$$\frac{F}{m} = \frac{ma}{m} \quad (2-16)$$

We use the horizontal line for division because it is clearer. Remember that

$$a \div b = \frac{a}{b} \quad (2-17)$$

We rearrange the symbols on the right hand side so that one m is on top of the other m .

$$\frac{F}{m} = \frac{m}{m}a \quad (2-18)$$

Any number divided by itself is always 1. For example $2 \div 2 = 1$. So we can say.

$$\frac{m}{m} = 1 \quad (2-19)$$

Now we can replace the $\frac{m}{m}$ on the right hand side with 1! This simplifies the equation.

$$\frac{F}{m} = a \quad (2-20)$$

Now we flip the equation around and put a on the left hand side and $\frac{F}{m}$ on the right hand side.

$$a = \frac{F}{m} \quad (2-21)$$

This tells us what the acceleration a is if we know the force, F and m mass!

Let's do some problems. The solutions are at the end of the book.

Problem 1

What is the force that gravity exerts on a 10 kilogram object? The acceleration of gravity is 10 m/sec.

Problem 2

Your arms can produce a force of 100 Newtons. What acceleration can you place on a 2 kg object?

Real World Problem

Principal Dancer Pedro has been dancing with his partner Akina whose mass is 40 kg all season. Akina has injured her ankle and cannot dance the Grande Pas de Deux in Don Quixote. Instead he must dance with Maria whose mass is 45 kg. The acceleration of gravity is 10 meters per second squared. Pedro can produce a force of 420 Newtons with his arms. Can he lift Maria?

What Have We Learned

In this chapter we learned how to manipulate an equation. We've learned about symbols and how they represent numbers.

CHAPTER 3

THE BATHROOM SCALE

Let us suppose we have a bathroom scale with a rotating needle. The force on the needle mechanism (you could also say the machine that operates the needle) is

$$F = k\theta \quad (3-1)$$

where F is the force on the needle, k is the spring constant and θ (the Greek letter called theta) is the angle of the needle. This equation is read as

force equals the spring constant times the angle of the needle

or

F equals k times theta

k is a constant associated with the mechanism and is determined by the design. This is the equation that tells us that if we apply the force, F to the mechanism the needle will move. How much will it move? We need to solve this equation for θ as follows

$$F = k\theta \quad (3-2)$$

$$\frac{F}{k} = \frac{k\theta}{k} \quad (3-3)$$

$$\frac{F}{k} = \frac{k}{k}\theta \quad (3-4)$$

$$\frac{k}{k} = 1 \quad (3-5)$$

$$\frac{F}{k} = \theta \quad (3-6)$$

$$\theta = \frac{F}{k} \quad (3-7)$$

So how does a scale work? When we step on a scale we apply the force of gravity

$$F = mg \quad (3-8)$$

where g is the acceleration of gravity and m is our mass. Notice that we use the same symbol F for force. What happens if we apply this force to the scale? We just need to substitute the force due to gravity into the equation for the scale. This is called a force balance. This is saying that the *force of gravity on the person standing on the scale equals the force on the spring*.

We wrote previously

$$\theta = \frac{F}{k} \quad (3-9)$$

We substitute the mg in Eq. 3-8 on the preceding page for the F in Eq. 3-9 and get

$$\theta = \frac{mg}{k} \quad (3-10)$$

Because F equals mg we can freely use mg in place of the F . This equation reads

the angle of the needle equals the mass of the person on the scale times the acceleration of gravity divided by the constant of the spring

Let's look at some numbers. If your mass is 40 kg, the acceleration of gravity is 10 meters/sec² (which you read as *10 meters per second squared*, we'll talk about the term squared in the next chapter) and our spring constant is 400 we get

$$\theta = \frac{40 \times 10}{400} \quad (3-11)$$

or 1! So the needle will move the unit of angle 1. As we discussed in the previous chapter the equation is a sort of machine in which you can put in numbers and get another number. Each symbol can represent *any number*.

Problem 1

Suppose that your dad stands on the scale and his mass is 80 kg. How far will the needle move?

Problem 2

Suppose you observe that the needle moves 3 units. What is the mass of the person standing on the scale? How did you figure this out? Write the equation you used to solve for m .

Real World Problem

Chief Engineer Kate at Consolidated Mechanisms comes to you and says that she doesn't think your spring supplier gave you the right spring. Its spring constant is supposed to be 400 but when an 80 kg person stood on the scale the needle moved 4 units. Unfortunately, you've shipped the scale and you are going to have many grumpy customers if she is right. Is Kate right? What is k ? Will your customers think they are too fat or skinny? How did you figure this out? Write the equation you used to solve for k .

What Have We Learned

In this chapter we learned how to equate two equations to get a new equation. We also practiced manipulating an equation to solve for a specific symbol. We've learned about springs. We also learned how a bathroom scale works.

CHAPTER 4

ANSWERS TO PROBLEMS

Chapter 2

Problem 1

Write the equation

$$F = ma \quad (4-1)$$

Putting in the numbers

$$F = 10 \times 10 \quad (4-2)$$

$$F = 100$$

Problem 2

Write the equation

$$F = ma \quad (4-3)$$

Solve for a

$$F = ma \quad (4-4)$$

$$\frac{F}{m} = \frac{ma}{m} \quad (4-5)$$

$$\frac{F}{m} = \frac{m}{m}a \quad (4-6)$$

$$\frac{m}{m} = 1 \quad (4-7)$$

$$\frac{F}{m} = a \quad (4-8)$$

$$a = \frac{F}{m} \quad (4-9)$$

Putting in numbers

$$a = \frac{100}{2} \quad (4-10)$$

so $a = 50$

Real World Problem

Write

$$F = ma \quad (4-11)$$

Inserting numbers

$$F = 45 \times 10 \quad (4-12)$$

His arms need to produce a force of 450 so the answer is no he won't be able to lift Maria. With Akina

$$F = 40 \times 10 \quad (4-13)$$

or $F = 400$ so he can lift Akina!

Chapter 3

Problem 1

Write the equation

$$\theta = \frac{mg}{k} \quad (4-14)$$

Putting in the numbers

$$\theta = \frac{80 \times 10}{400} \quad (4-15)$$

$$\theta = 2$$

Problem 2

Write the equation

$$\theta = \frac{mg}{k} \quad (4-16)$$

Solve for m

$$\theta \frac{k}{g} = \frac{mg}{k} \frac{k}{g} \quad (4-17)$$

$$m = \theta \frac{k}{g} \quad (4-18)$$

Substitute in the numbers

$$m = 3 \times \frac{400}{10} \quad (4-19)$$

$$m = 120.$$

Real World Problem

We need to solve for the spring constant k . Write

$$\theta = \frac{mg}{k} \quad (4-20)$$

Multiply both sides by k

$$\theta k = \frac{mg}{k} k \quad (4-21)$$

Cancel the k 's on the right-hand-side and divide both sides by θ .

$$k = \frac{mg}{\theta} \quad (4-22)$$

Substitute in the numbers

$$k = \frac{80 \times 10}{4} \quad (4-23)$$

$k = 200$. Kate is right. She isn't your chief engineer for nothing. So are the weights people seeing too high or too low? Now we plug in the numbers.

$$\theta = \frac{mg}{k} \quad (4-24)$$

k is supposed to be 400 by instead it is 200. For the same mass θ will be twice as big. For example for our 80 kg person

$$\theta = \frac{80 \times 10}{200} = 4 \quad (4-25)$$

for the wrong spring or

$$\theta = \frac{80 \times 10}{400} = 2 \quad (4-26)$$

for the right spring. So since θ is showing twice as large a value people will think they are twice as heavy.

